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Theoretical Study on the Interaction Between Constant-Pressure Specific Heat and Nonequilibrium Phase Change Process in Two-Phase Flow

Wen-Qiang Lu · Kai Xu

Abstract In two-phase flow, the constant-pressure specific heat of a mixture correlates with the flow and the heat transfer processes. In this paper, the air-water-vapor system is taken as an example, and the behavior of the constant-pressure specific heat during a nonequilibrium phase change process in a two-phase flow system is deduced using the theory of two-phase flow and thermophysics; corresponding calculations are employed to the actual two-phase flow process. The results show that the flow and the phase change heat transfer processes determine the variation and magnitude of the specific heat. Vice versa, the specific heat affects the flow and the phase change heat transfer processes.

Keywords Constant-pressure specific heat · Nonequilibrium phase change · Two-phase flow

1 Introduction

The constant-pressure specific heat is a very important thermophysical property in two-phase flow. The phase change process of the dispersed phase has an effect on the constant-pressure specific heat of the mixture fluid, and thus affects the heat transfer and flow characteristics of the fluid. However, a correct calculation of the specific heat has been neglected sometimes. The mass average specific heat of two-phase substances has been used in the inaccurate computation of two-phase flow and phase change heat transfer. The investigation of a correct formula of the specific heat with the phase change process is necessary. A formula for the constant-pressure specific

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heat with an equilibrium phase change process was reported in [1]. However, in actual two-phase flow, it is a nonequilibrium process because of the different velocities and temperatures between the two phases. The constant-pressure specific heat formula is often based on some assumptions and experience, which lack a theoretical basis [2,3]. A formula for the constant-pressure specific heat with a nonequilibrium phase change process was deduced in [4] and successfully applied to the flow and the heat transfer processes of the latent functionally thermal fluid [5]. However, it should be noted that in [4,5] the phase change materials were packed in micro-capsules, whose sizes remained constant in the phase change process.

In this paper, considering the size variation of the phase change particles, a formula for the constant-pressure specific heat during a nonequilibrium phase change process in a two-phase flow system is deduced strictly on the basis of the theory of two-phase flow and thermophysics. Employing the formula, a numerical simulation of the flow and the heat transfer processes of air-water-vapor two-phase flow in a circular tube is carried out, and the interactions between the specific heat and the droplet evaporation process are investigated. The methods and results can also be extended to other similar two-phase flow systems.

2 Theoretical Derivation of the Constant-Pressure Specific Heat

Considering the process of droplet evaporation, the water vapor and air are mixed uniformly, and the dissolvability of air in water and the interfacial force are neglected. In the following derivation, subscript p represents the droplet, subscript f represents the fluid phase which consists of air and water vapor, and subscripts a, m, and v represent air, the two-phase mixture, and the water vapor, respectively. K_i and h_i represent the mass fraction and the specific enthalpy of phase i, respectively.

On the basis of the law of mass conservation:

$$\mathrm{d}K_\mathrm{p} + \mathrm{d}K_\mathrm{v} + \mathrm{d}K_\mathrm{a} = 0 \tag{1}$$

$$\mathrm{d}K_\mathrm{a} = 0, \mathrm{d}K_\mathrm{p} + \mathrm{d}K_\mathrm{v} = 0 \tag{2}$$

The relation between the enthalpy of the two-phase mixture and each phase is as follows:

$$h_\mathrm{m} = \sum_i K_i h_i = K_\mathrm{a} h_\mathrm{a} + K_\mathrm{p} h_\mathrm{p} + K_\mathrm{v} h_\mathrm{v} \tag{3}$$

Taking a derivative of Eq. 3, we have

$$\mathrm{d}h_\mathrm{m} = \sum_i K_i \mathrm{d}h_i + \sum_i h_i \mathrm{d}K_i \tag{4}$$

$$\sum_i K_i \mathrm{d}h_i = \sum_i K_i \left[\left(\frac{\partial h_i}{\partial T_i} \right)_{p_i} \mathrm{d}T_i + \left(\frac{\partial h_i}{\partial p_i} \right)_{T_i} \mathrm{d}p_i \right] \tag{5}$$

where T and p represent the temperature and pressure, respectively.

A local equilibrium of nonequilibrium thermodynamics is assumed, in which a finite-size nonequilibrium macroscopic multiphase system is separated into elements that are small enough on a macroscopic scale and large enough on a microscopic scale. So every element can be considered as a macroscopic thermodynamic subsystem, and its variation of thermodynamic properties is small. The same micro-size variation of temperature occurs in every phase of the system during the phase change process: $\mathrm{d}T_i = \mathrm{d}T_\mathrm{m} = \mathrm{d}T$.

$$\sum_i K_i \mathrm{d}h_i = \sum_i K_i c_{pi} \mathrm{d}T + \sum_i K_i \left(\frac{\partial h_i}{\partial p_i}\right)_{T_i} \mathrm{d}p_i \tag{6}$$

where c_{pi} is the constant-pressure specific heat of phase i. According to Eq. 2, we have

$$\sum_i h_i \mathrm{d}K_i = \left(h_\mathrm{p} - h_\mathrm{v}\right) \mathrm{d}K_\mathrm{p} = \Delta h_\mathrm{m} \mathrm{d}K_\mathrm{p} \tag{7}$$

where Δh_m is the enthalpy variation in the phase change process. Substituting Eqs. 6 and 7 into Eq. 4, we have

$$\mathrm{d}h_\mathrm{m} = \sum_i K_i c_{pi} \mathrm{d}T + \sum_i K_i \left(\frac{\partial h_i}{\partial p_i}\right)_{T_i} \mathrm{d}p_i + \Delta h_\mathrm{m} \mathrm{d}K_\mathrm{p} \tag{8}$$

Based on two-phase flow theory, we have

$$\rho_\mathrm{m} = Z_\mathrm{p}\rho_\mathrm{p} + Z_\mathrm{f}\rho_\mathrm{f} \tag{9}$$

where ρ and Z are the density and volume fraction, respectively.

Assume that the vapors are diffused so rapidly that they can be mixed with air completely, i.e., the volume of the fluid phase = the volume of the air = the volume of the vapor, and then we have

$$Z_\mathrm{f} = Z_\mathrm{a} = Z_\mathrm{v}, \quad Z_\mathrm{f} + Z_\mathrm{p} = 1 \tag{10}$$

$$\rho_\mathrm{m} = \rho_\mathrm{a} Z_\mathrm{f}/K_\mathrm{a} = \rho_\mathrm{v} Z_\mathrm{f}/K_\mathrm{v} = \rho_\mathrm{f} Z_\mathrm{f}/K_\mathrm{f} = \rho_\mathrm{p} Z_\mathrm{p}/K_\mathrm{p} \tag{11}$$

Differentiating Eqs. 9–11, respectively, we have the following equations:

$$\mathrm{d}\rho_\mathrm{m} = Z_\mathrm{p}\mathrm{d}\rho_\mathrm{p} + \rho_\mathrm{p}\mathrm{d}Z_\mathrm{p} + Z_\mathrm{f}\mathrm{d}\rho_\mathrm{f} + \rho_\mathrm{f}\mathrm{d}Z_\mathrm{f} \tag{12}$$

$$\mathrm{d}Z_\mathrm{f} + \mathrm{d}Z_\mathrm{p} = 0 \tag{13}$$

$$\mathrm{d}\rho_\mathrm{m} = \frac{\rho_\mathrm{a}}{K_\mathrm{a}}\mathrm{d}Z_\mathrm{f} + \frac{Z_\mathrm{f}}{K_\mathrm{a}}\mathrm{d}\rho_\mathrm{a} \tag{14}$$

From Eqs. 12–14, the following equation is obtained by eliminating $\mathrm{d}\rho_\mathrm{m}$:

$$\mathrm{d}Z_\mathrm{f} = \frac{1}{\frac{\rho_\mathrm{a}}{K_\mathrm{a}} + \rho_\mathrm{p} - \rho_\mathrm{f}} \left(Z_\mathrm{p}\mathrm{d}\rho_\mathrm{p} + Z_\mathrm{f}\mathrm{d}\rho_\mathrm{f} - \frac{Z_\mathrm{f}}{K_\mathrm{a}}\mathrm{d}\rho_\mathrm{a}\right) \tag{15}$$

Substituting Eq. 15 into Eq. 12 and dividing the result by ρ_m, we have the following equation:

$$\frac{\mathrm{d}\rho_\mathrm{m}}{\rho_\mathrm{m}} = \frac{Z_\mathrm{p}}{Z_\mathrm{f}}\frac{1}{A}\mathrm{d}\rho_\mathrm{p} + \frac{1}{A}\mathrm{d}\rho_\mathrm{f} + \left(\frac{1}{\rho_\mathrm{a}} - \frac{1}{K_\mathrm{a}}\frac{1}{A}\right)\mathrm{d}\rho_\mathrm{a} \tag{16}$$

where

$$A = \frac{\rho_\mathrm{a}}{K_\mathrm{a}} + \rho_\mathrm{p} - \rho_\mathrm{f} \tag{17}$$

On the basis of the above assumption, the following equation is obtained:

$$\rho_\mathrm{f} = \rho_\mathrm{a} + \rho_\mathrm{v} \tag{18}$$

Substituting Eq. 18 into Eq. 16, and from Eq. 11, we obtain the following equation:

$$\frac{\mathrm{d}\rho_\mathrm{m}}{\rho_\mathrm{m}} = Z_\mathrm{p}\frac{\mathrm{d}\rho_\mathrm{p}}{\rho_\mathrm{p}} + \frac{Z_\mathrm{f}}{\rho_\mathrm{p}}\mathrm{d}\rho_\mathrm{a} + \frac{Z_\mathrm{f}}{\rho_\mathrm{p}}\mathrm{d}\rho_\mathrm{v} + \left(\frac{1}{\rho_\mathrm{a}} - \frac{Z_\mathrm{f}}{K_\mathrm{a}\rho_\mathrm{p}}\right)\mathrm{d}\rho_\mathrm{a} \tag{19}$$

From Eq. 11, we obtain the following equation:

$$\mathrm{d}K_\mathrm{p} = K_\mathrm{p}\left(\frac{\mathrm{d}Z_\mathrm{p}}{Z_\mathrm{p}} - \frac{\mathrm{d}\rho_\mathrm{m}}{\rho_\mathrm{m}} + \frac{\mathrm{d}\rho_\mathrm{p}}{\rho_\mathrm{p}}\right) \tag{20}$$

Substituting Eq. 19 into Eq. 20, we obtain

$$\mathrm{d}K_\mathrm{p} = K_\mathrm{p}\left[\frac{\mathrm{d}Z_\mathrm{p}}{Z_\mathrm{p}} + \left(1 - Z_\mathrm{p}\right)\frac{\mathrm{d}\rho_\mathrm{p}}{\rho_\mathrm{p}}\right] - K_\mathrm{v}Z_\mathrm{p}\frac{\mathrm{d}\rho_\mathrm{v}}{\rho_\mathrm{v}} + \left(Z_\mathrm{p} - K_\mathrm{p} - K_\mathrm{a}Z_\mathrm{p}\right)\frac{\mathrm{d}\rho_\mathrm{a}}{\rho_\mathrm{a}} \tag{21}$$

The relation between $\mathrm{d}Z_\mathrm{p}/Z_\mathrm{p}$ and the mass fraction is deduced as follows: according to Eq. 11, we have $\frac{Z_\mathrm{p}\rho_\mathrm{p}}{K_\mathrm{p}} = \frac{Z_\mathrm{f}\rho_\mathrm{a}}{K_\mathrm{a}} = \rho_\mathrm{m} = \frac{(1-Z_\mathrm{p})\rho_\mathrm{a}}{K_\mathrm{a}}$, and the following equation is obtained:

$$Z_\mathrm{p} = \frac{\rho_\mathrm{a}/K_\mathrm{a}}{\rho_\mathrm{a}/K_\mathrm{a} + \rho_\mathrm{p}/K_\mathrm{p}} \tag{22}$$

Letting $M = \rho_\mathrm{a}/K_\mathrm{a} + \rho_\mathrm{p}/K_\mathrm{p}$, and differentiating Eq. 22, we obtain

$$\frac{\mathrm{d}Z_\mathrm{p}}{Z_\mathrm{p}} = \left(1 - Z_\mathrm{p}\right)\frac{\mathrm{d}\rho_\mathrm{a}}{\rho_\mathrm{a}} - \frac{1}{M}\frac{\rho_\mathrm{p}}{K_\mathrm{p}}\frac{\mathrm{d}\rho_\mathrm{p}}{\rho_\mathrm{p}} + \frac{1}{M}\frac{\rho_\mathrm{p}}{K_\mathrm{p}^2}\mathrm{d}K_\mathrm{p} \tag{23}$$

Substituting Eq. 23 into Eq. 21, and from Eq. 11, we obtain

$$\mathrm{d}K_\mathrm{p} = K_\mathrm{v}\frac{\mathrm{d}\rho_\mathrm{a}}{\rho_\mathrm{a}} - K_\mathrm{v}\frac{\mathrm{d}\rho_\mathrm{v}}{\rho_\mathrm{v}} \tag{24}$$

Since the phase density of every phase is a function of pressure and temperature, we have

$$\mathrm{d}\rho_i = \left(\frac{\partial \rho_i}{\partial T_i}\right)_{p_i} \mathrm{d}T_i + \left(\frac{\partial \rho_i}{\partial p_i}\right)_{T_i} \mathrm{d}p_i \tag{25}$$

It should be noted that the change of the vapor density is not only caused by the effect of thermal expansion, but also by the evaporation of droplets, diffusing the vapor into the air. This is the important difference between air-water-vapor two-phase flow and the latent functionally thermal fluid with microencapsulated phase change materials [4]. Thus

$$\mathrm{d}\rho_\mathrm{v} = \left(\frac{\partial \rho_\mathrm{v}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}} \mathrm{d}T_\mathrm{v} + \left(\frac{\partial \rho_\mathrm{v}^\mathrm{evp}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}} \mathrm{d}T_\mathrm{v} + \left(\frac{\partial \rho_\mathrm{v}}{\partial p_\mathrm{v}}\right)_{T_\mathrm{v}} \mathrm{d}p_\mathrm{v} + \left(\frac{\partial \rho_\mathrm{v}^\mathrm{evp}}{\partial p_\mathrm{v}}\right)_{T_\mathrm{v}} \mathrm{d}p_\mathrm{v} \tag{26}$$

where $\left(\frac{\partial \rho_\mathrm{v}^\mathrm{evp}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}} \mathrm{d}T_\mathrm{v}$ represents the evaporation effect.

$$\left(\frac{\partial \rho_\mathrm{v}^\mathrm{evp}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}} \mathrm{d}T_\mathrm{v} = \frac{\rho_\mathrm{v}}{\rho_\mathrm{v}} \left(\frac{\partial \rho_\mathrm{v}^\mathrm{evp}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}} \mathrm{d}T_\mathrm{v} = \frac{\rho_\mathrm{v}}{m_\mathrm{v}} \left(\frac{\partial m_\mathrm{v}^\mathrm{evp}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}} \mathrm{d}T_\mathrm{v} = \frac{\rho_\mathrm{v}}{K_\mathrm{v}} \left(\frac{\partial K_\mathrm{v}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}} \mathrm{d}T_\mathrm{v} \tag{27}$$

Then we have the following equation:

$$\begin{aligned} \mathrm{d}K_\mathrm{p} &= K_\mathrm{v}\frac{\mathrm{d}\rho_\mathrm{a}}{\rho_\mathrm{a}} - K_\mathrm{v}\frac{\mathrm{d}\rho_\mathrm{v}}{\rho_\mathrm{v}} = K_\mathrm{v}\left[-\beta_\mathrm{a}\mathrm{d}T + \beta_\mathrm{v}\mathrm{d}T - \frac{1}{K_\mathrm{v}}\left(\frac{\partial K_\mathrm{v}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}} \mathrm{d}T\right] \\ &\quad + K_\mathrm{v}\left[\frac{1}{\rho_\mathrm{a}}\left(\frac{\partial \rho_\mathrm{a}}{\partial p_\mathrm{a}}\right)_{T_\mathrm{a}} \mathrm{d}p_\mathrm{a} - \frac{1}{\rho_\mathrm{v}}\left(\frac{\partial \rho_\mathrm{v}}{\partial p_\mathrm{v}}\right)_{T_\mathrm{v}} \mathrm{d}p_\mathrm{v} - \frac{1}{\rho_\mathrm{v}}\left(\frac{\partial \rho_\mathrm{v}^\mathrm{evp}}{\partial p_\mathrm{v}}\right)_{T_\mathrm{v}} \mathrm{d}p_\mathrm{v}\right] \end{aligned} \tag{28}$$

where β_i is the expansion coefficient at constant pressure of phase i:

$$\beta_i = \frac{(\partial v_i/\partial T_i)_{p_i}}{v_i} = -\frac{(\partial \rho_i/\partial T_i)_{p_i}}{\rho_i} \tag{29}$$

Substituting Eq. 28 into Eq. 8 and according to the definition of the specific heat, the constant-pressure specific heat of the two-phase mixture with a nonequilibrium phase change process can be expressed as

$$\begin{aligned} c_{p\mathrm{m}} &= \sum_i K_i c_{pi} + \Delta h_\mathrm{m} K_\mathrm{v}\left[\beta_\mathrm{v} - \beta_\mathrm{a} - \frac{1}{K_\mathrm{v}}\left(\frac{\partial K_\mathrm{v}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}}\right] \\ &= c_{p\mathrm{m}0} + \Delta h_\mathrm{m} K_\mathrm{v}\left[\beta_\mathrm{v} - \beta_\mathrm{a} - \frac{1}{K_\mathrm{v}}\left(\frac{\partial K_\mathrm{v}}{\partial T_\mathrm{v}}\right)_{p_\mathrm{v}}\right] \end{aligned} \tag{30}$$

where $c_{p\mathrm{m}0}$ is the mass average constant-pressure specific heat of two-phase substances.

3 Numerical Models and Methods

In order to verify the formula for the constant-pressure specific heat, air-water-vapor two-phase flow in a circular tube is used as an example to study the specific heat variation in the phase change process. During the flow process, the mixture of air and droplets is heated by a constant wall heat flux, and the moving droplets are continually vaporized. The tube length is $L = 924\,\mathrm{mm}$, and the inner diameter is $D = 13.2\,\mathrm{mm}$. The mass fraction of the droplets at the inlet is 0.5 %, and the initial diameter is $d_{\mathrm{p}0} = 15.6\,\mu\mathrm{m}$. The mass flow rate is $7.58\,\mathrm{g}\cdot\mathrm{s}^{-1}$ and Re = 39,360. The details of the experiment are given in [6]. In the experiment, in the area of about $0 < L/D < 10$, there existed a liquid film near the wall due to the droplet deposition which is not considered in this paper for simplicity.

3.1 Fluid Equations

For the fully developed turbulent pipe flow, as a one-seventh-power law for the velocity profile in the turbulent boundary layer is assumed,

$$u_{\mathrm{f}} = u_{\mathrm{fm}}\,(1 - r/R)^{1/7}\,, \quad v_{\mathrm{f}} = 0 \tag{31}$$

where u_{f} and v_{f} are the axial and radial velocities of the fluid phase, respectively, u_{fm} is the maximum velocity, and R is the tube radius.

The energy equation of the fluid phase is

$$(1 - Z_{\mathrm{p}})\rho_{\mathrm{f}} u_{\mathrm{f}} c_{p\mathrm{f}} \frac{\partial T_{\mathrm{f}}}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r c_{p\mathrm{f}} \left(\frac{\mu}{Pr} + \frac{\mu_{\mathrm{T}}}{Pr_{\mathrm{T}}} \right) \frac{\partial T_{\mathrm{f}}}{\partial r} \right] + S \tag{32}$$

where μ is the dynamic viscosity, Pr is the Prandtl number, and Pr_{T} is an empirical constant. μ_{T} is the turbulent viscosity which is given by Prandtl mixing length theory, where l_{m} is the mixing length,

$$\mu_{\mathrm{T}} = \rho_{\mathrm{f}} l_{\mathrm{m}}^2 \left| \frac{\partial u_{\mathrm{f}}}{\partial r} \right| \tag{33}$$

The source term S represents heat exchange between the droplets and the fluid:

$$S = 3\alpha_{\mathrm{p}} Z_{\mathrm{p}} \left(T_{\mathrm{p}} - T_{\mathrm{f}} \right) / r_{\mathrm{p}} \tag{34}$$

where α_{p} is the heat convection coefficient between the droplets and fluid and r_{p} is the radius of the droplet.

3.2 Droplet Equations

Since the droplet diameter is very small and the magnitude of the Stokes number is about 10^{-4}, the droplet velocity can be considered the same as the fluid. Assuming the inner temperature of a droplet is uniform, the energy equation of a single droplet is

$$m_{\mathrm{p}} c_{p\mathrm{p}} \frac{\mathrm{d}T_{\mathrm{p}}}{\mathrm{d}t} - 4\pi r_{\mathrm{p}}^2 \rho_{\mathrm{p}} H \frac{\mathrm{d}r_{\mathrm{p}}}{\mathrm{d}t} = 4\pi r_{\mathrm{p}}^2 q \tag{35}$$

where m_{p} is the mass of the droplet and H is the latent heat of vaporization. q is the heat transported to the droplet per unit surface area.

The rate of change of the droplet radius is [7]

$$\frac{\mathrm{d}r_{\mathrm{p}}}{\mathrm{d}t} = -\frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{p}}} \frac{D_{\mathrm{f}}}{2r_{\mathrm{p}}} Sh \ln(1 + B_{\mathrm{M}}), \tag{36}$$

and heat convection between the droplet and fluid is

$$q = \frac{k_{\mathrm{f}}(T_{\mathrm{f}} - T_{\mathrm{p}})}{2r_{\mathrm{p}}} Nu \frac{\ln(1 + B_{\mathrm{M}})}{B_{\mathrm{M}}} \tag{37}$$

where k_{f} and D_{f} are the thermal conductivity and mass diffusivity of the fluid phase, respectively. Nu is the Nusselt number; Sh is the Sherwood number, and B_{M} is the Spalding mass transfer number [8].

3.3 Mixture Fluid Equations

The energy equation of the fluid mixture is

$$\rho_{\mathrm{m}} u_{\mathrm{f}} c_{p\mathrm{m}} \frac{\partial T_{\mathrm{m}}}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left[r c_{p\mathrm{m}} \left(\frac{\mu}{Pr} + \frac{\mu_{\mathrm{T}}}{Pr_{\mathrm{T}}} \right) \frac{\partial T_{\mathrm{m}}}{\partial r} \right] \tag{38}$$

Compared to the fluid phase energy equation, Eq. 32, there is no source term resulting from the heat transport between the fluid and droplets. The dynamic viscosity, Prandtl number, and the turbulent viscosity of the mixture are assumed to be the same as that of the fluid phase due to the small mass fraction of the droplets.

3.4 Numerical Method

The detailed calculation steps are as follows:

(i) Give the total temperature of the fluid phase $T_{\mathrm{f}}^{\mathrm{old}}$ and the initial temperature of the droplets;
(ii) Solve Eqs. 35–37 to get the droplet temperature T_{p};

(iii) Substitute T_p into Eq. 34 to calculate the source term S and then solve Eq. 32 to get the new fluid phase temperature T_f^{new}, which is then compared to T_f^{old}. If the error between them does not satisfy the required precision, then a relaxation step is used to get a new temperature and return to step (ii); otherwise, the iterative calculation is ended;

(iv) Solve Eq. 30 to obtain the constant-pressure specific heat of the mixture, and then substitute it into Eq. 38 to get the mixture fluid temperature T_m.

4 Results and Discussion

4.1 Verification of the Numerical Model

Figure 1 depicts the calculated wall temperature compared to the experimental data [6]. The wall heat flux values q_w of the three tests are 11.50, 13.92, and 17.84 kW · m^{-2}. As shown in the figure, in the area without droplet deposition, better agreement between numerical results and experimental data is obtained, showing that the present numerical model and method are reliable.

4.2 Numerical Results of the Constant-Pressure Specific Heat

Figure 2 depicts the axial variations of the ratio c_{pm}/c_{pm0} at distances $0.2R$, $0.4R$, $0.6R$, and $0.8R$ from the axis. As shown in the figure, there are evident differences between the specific heats, and the variations of the ratio at different places are also shown. This is mainly because there are different complex phase change heat transfer

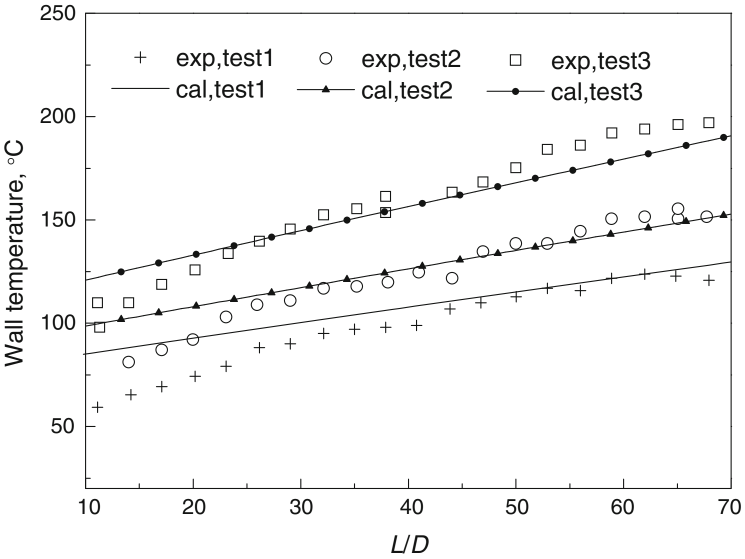

Fig. 1 Comparison of calculated and experimental wall temperatures [6]

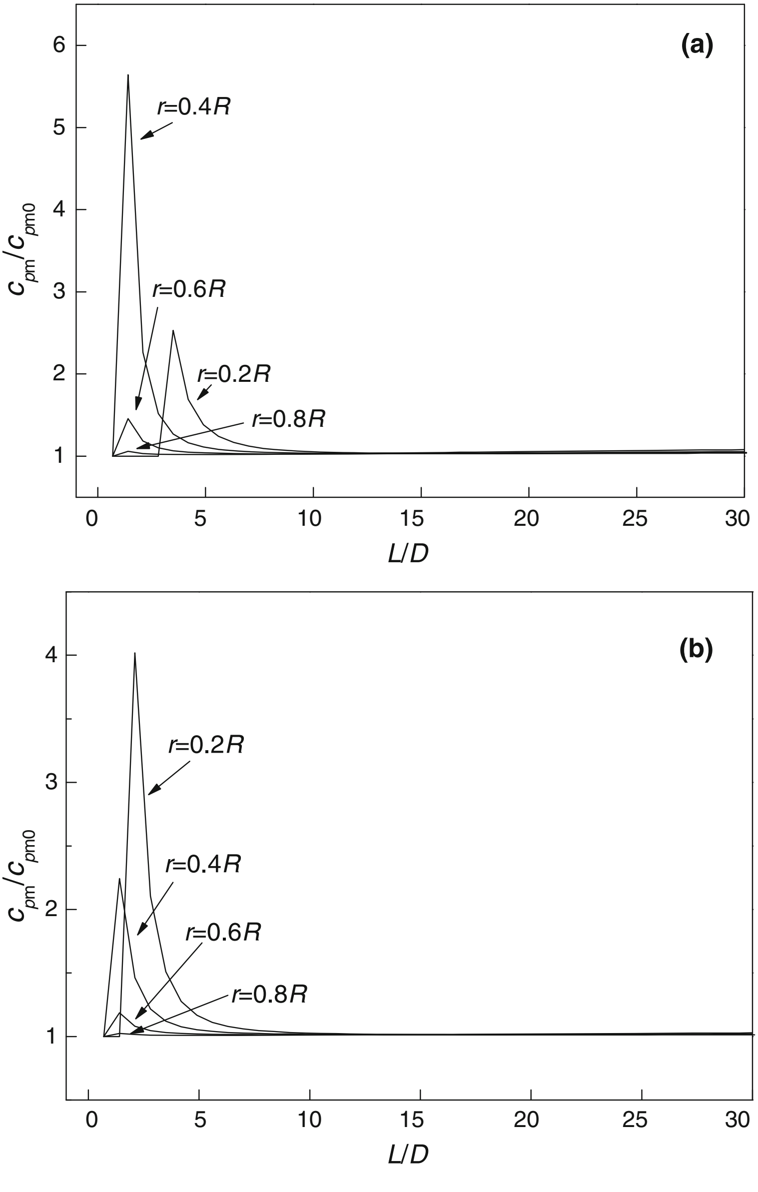

Fig. 2 Variations of the ratio between the specific heat with nonequilibrium phase change and mass average value ($q_{\mathrm{w}} = 17.84\,\mathrm{kW}\cdot\mathrm{m}^{-2}$): (**a**) $d_{\mathrm{p0}} = 15.6\,\mu\mathrm{m}$ and (**b**) $d_{\mathrm{p0}} = 25\,\mu\mathrm{m}$

and flow processes at different places. Comparing Fig. 2a and b, we can see that the smaller the particle is, the higher the ratio peak value is. Therefore, it can be concluded that the evaporation process directly affects the variation of the constant-pressure specific heat, which is the important characteristic of two-phase mixture flow with a phase change process.

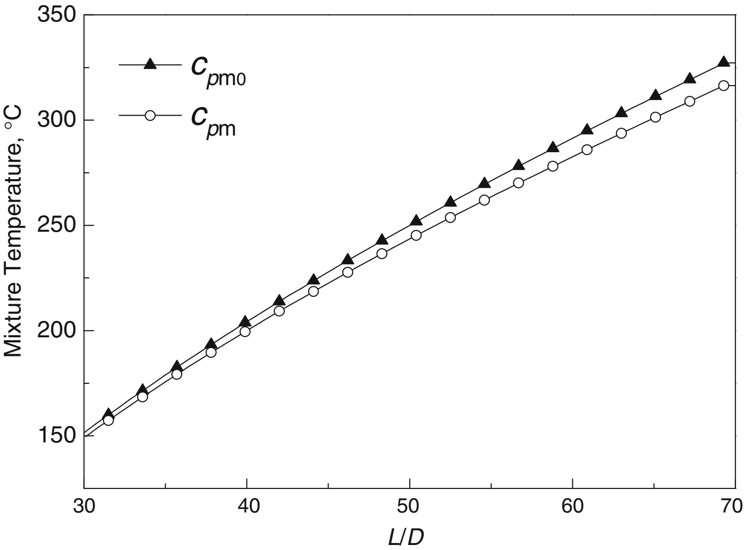

Fig. 3 Comparison of the mixture temperature along the symmetry axis calculated with $c_{p\mathrm{m}}$ and $c_{p\mathrm{m}0}$; Re = 15,000

4.3 Mixture Temperatures Calculated with $c_{p\mathrm{m}}$ and $c_{p\mathrm{m}0}$

Figure 3 depicts the mixture temperature along the symmetry axis calculated with $c_{p\mathrm{m}}$ and $c_{p\mathrm{m}0}$, where $q_\mathrm{w} = 17.84\,\mathrm{kW}\cdot\mathrm{m}^{-2}$ and $d_{\mathrm{p}0} = 15.6\,\mu\mathrm{m}$. As shown in the figure, the mixture temperature calculated with $c_{p\mathrm{m}0}$ is higher than that calculated with $c_{p\mathrm{m}}$. This shows that the specific heat directly affects the flow and the phase change heat transfer processes.

4.4 Difference Between the Equilibrium and Nonequilibrium Phase Change Processes

In order to compare the difference in the constant-pressure specific heat between the nonequilibrium and equilibrium phase change processes, a formula from the literature [1] is selected to calculate the specific heat of the equilibrium phase change process:

$$\tilde{c}_{p\mathrm{m}} = \sum_i K_i c_{pi} + \frac{K_\mathrm{v}}{p_\mathrm{a}} \frac{H^2}{\tilde{T}\Delta v} \tag{39}$$

where $\tilde{T}$ is the equilibrium temperature. Δv is the specific volume difference between water vapor and liquid water, $\Delta v = v_\mathrm{v} - v_\mathrm{p}$. In the calculation, $\tilde{T}$ is chosen as the fluid temperature. Figure 4 shows comparisons of the mixture temperature between nonequilibrium and equilibrium phase change processes at $r = 0.2R$. As shown in the figure, the temperature of the equilibrium process is higher than that of the

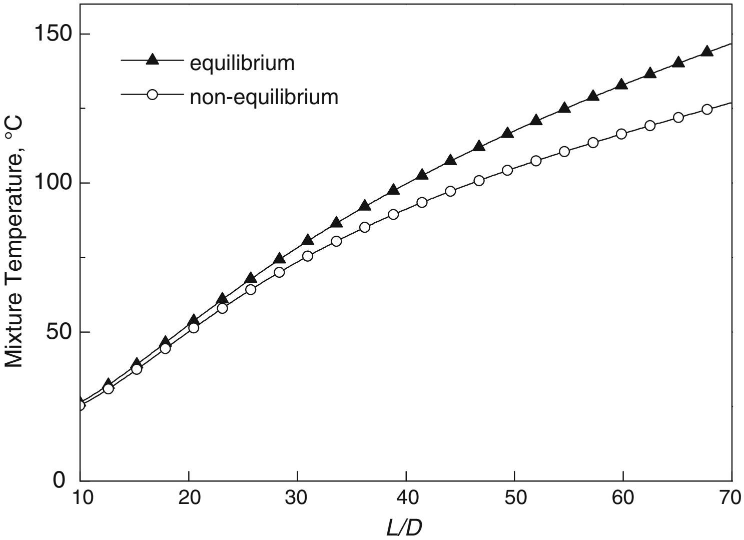

Fig. 4 Comparison of equilibrium and nonequilibrium phase change process at $r = 0.2R$; $q_{\mathrm{w}} = 17.84\,\mathrm{kW \cdot m^{-2}}$; and $d_{\mathrm{p}0} = 15.6\,\mu\mathrm{m}$

nonequilibrium process, and the difference increases gradually as the evaporation process proceeds.

5 Conclusions

In this paper, the air-water-vapor system is taken as an example, and a formula for the constant-pressure specific heat during a nonequilibrium phase change process in the two-phase flow system is deduced using the theory of two-phase flow and thermophysics; corresponding calculations are employed for the actual two-phase flow process.

The results show that the droplet position in the flow field and the droplet size affect the magnitude and variation of the specific heat. This shows that the constant-pressure specific heat is obviously affected by the two-phase flow and the phase change heat transfer processes. Simultaneously, numerical results show the different values of the mixture fluid temperatures calculated from $c_{p\mathrm{m}}$ and $c_{p\mathrm{m}0}$. This shows that the specific heat has a direct effect on the two-phase flow and the phase change heat transfer processes.

The numerical results also show that the mixture fluid temperatures of the nonequilibrium process are different from those of the equilibrium process. This should be considered in specific heat calculations of an actual two-phase flow system with a nonequilibrium phase change process.

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